Eur. Phys. J. A **18**, 499–502 (2003)
DOI 10.1140/epja/i2002-10269-3

THE EUROPEAN PHYSICAL JOURNAL A

Quark mass dependence of the nuclear forces

E. Epelbaum[1,a], Ulf-G. Meißner[2,3], and W. Glöckle[1]

[1] Ruhr-Universität Bochum, Institut für Theoretische Physik II, D-44870 Bochum, Germany
[2] Karl-Franzens-Universität Graz, Institut für Theoretische Physik A-8010 Graz, Austria
[3] Forschungszentrum Jülich, Institut für Kernphysik (Theorie), D-52425 Jülich, Germany

Received: 30 September 2002 /
Published online: 22 October 2003 –

Abstract. We investigate the behavior of the nuclear force as a function of the light-quark masses m_q in the framework of chiral effective field theory at next-to-leading order. The unknown m_q-dependent short-range contribution is estimated by means of dimensional analysis. We calculate various observables for different values of m_q. We found no new bound states and a larger deuteron binding energy, $B_D^{\mathrm{CL}} = 9.6 \pm 1.9^{+1.8}_{-1.0}$ MeV, in the chiral limit.

PACS. 11.30.Rd Chiral symmetries – 13.75.Cs Nucleon-nucleon interactions (including antinucleons, deuterons, etc.) – 21.30.Cb Nuclear forces in vacuum – 21.30.Fe Forces in hadronic systems and effective interactions

1 Introduction

Chiral Perturbation Theory (CHPT) is a well-established model-independent and systematic tool for calculating low-energy properties of hadronic systems. It is based upon the approximate and spontaneously broken chiral symmetry of QCD. Starting from the most general chiral invariant effective Lagrangian for Goldstone bosons (pions in the two-flavor case of light up and down quarks) and matter fields (nucleons, Δ-excitations, ...) low-energy S-matrix elements can be calculated via simultaneous expansion in the low external momenta and quark mass (or, equivalently, pion mass[1]). If two and more nucleons are considered, the interaction becomes too strong to be treated perturbatively and an additional nonperturbative resummation of the amplitude is necessary. Since the absolute values of the running up and down quark masses at the scale 1 GeV [1] $m_u \simeq 5$ MeV, $m_d \simeq 9$ MeV are rather small, one expects that hadronic properties at low energy do not change strongly in the chiral limit of (CL) $M_\pi \to 0$. This feature is crucial for the chiral expansion to make sense and is certainly true for the π and πN systems, where the interaction becomes arbitrarily weak in the CL and for vanishing external momenta. The purpose of this work is to look at the NN system in the CL (and, in general, for values of the pion mass different from the physical one), which is much more complicated due to the nonperturbative aspect. We stress that the question about the M_π-dependence of the nuclear force is not only of academic interest, but also of practical use for interpolating results from the lattice gauge theory, see [2] for more discussion. For example, the S-wave scattering lengths have been calculated recently on the lattice using the quenched approximation [3]. Another interesting application is related to imposing bounds on the time-dependence of fundamental couplings from the two-nucleon sector, as discussed in [4].

2 $\mathbf{M_\pi}$-dependence of the nuclear force

A convenient way of (nonperturbative) evaluation of the S-matrix elements for nucleon-nucleon (NN) interactions from the chiral effective Langangian suggested by Weinberg [5] is to apply CHPT methods to derive the effective NN potential. One can then calculate observables by solving the appropriate Lippmann-Schwinger equation. In [6] we have introduced a scheme, which leads to an energy-independent and Hermitian NN potential and is based upon the method of unitary transformation [7]. At leading order (LO) the potential is given by the (static) one-pion exchange (OPE) and two contact interactions without derivatives:

$$V^{\mathrm{LO}} = -\frac{1}{4}\frac{g_{\pi N}^2}{m_N^2}\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2\frac{(\boldsymbol{\sigma}_1\cdot\boldsymbol{q})(\boldsymbol{\sigma}_2\cdot\boldsymbol{q})}{\boldsymbol{q}^2+M_\pi^2}+C_S+C_T\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2, \quad (1)$$

where $g_{\pi N}$ is the pion-nucleon coupling constant, m_N the nucleon mass, $\boldsymbol{q}$ the nucleon momentum transfer, $\boldsymbol{\sigma}$ ($\boldsymbol{\tau}$)

---

[a] e-mail: `Evgeni.Epelbaum@tp2.ruhr-uni-bochum.de`

[1] In the isospin limit with the quark masses $m_u = m_d = \hat{m}$ one finds for the pion mass M_π: $M_\pi^2 = 2\hat{m}B(1+\mathcal{O}(\hat{m}))$, where B is a constant.

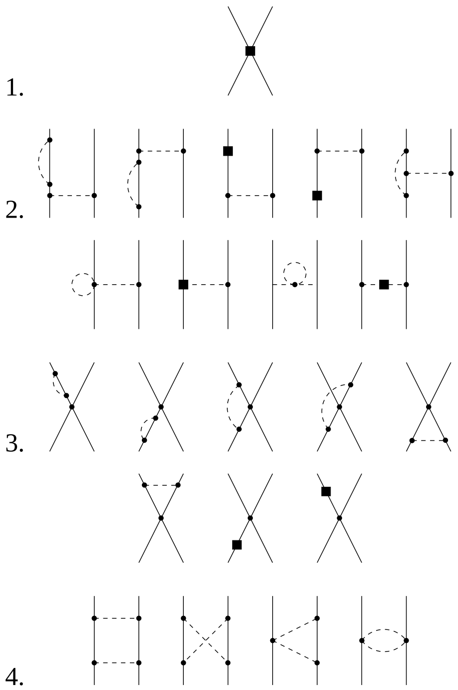

Fig. 1. NLO corrections to the NN potential. The solid (dashed) lines refer to nucleons (pions). The heavy dots denote the leading vertices, while the solid rectangles represent vertices of higher chiral dimension as defined in [6,8].

refer to spin (isospin) Pauli matrices and $C_{S,T}$ denote the low-energy constants (LECs). At next-to-leading order (NLO) the correction have to be taken into account from:

1. contact terms with two derivatives or one M_π^2-insertion,
2. renormalization of the OPE,
3. renormalization of the contact terms,
4. two-pion exchange (TPE).

The corresponding diagrams are depicted in fig. 1 in symbolic form, *i.e.* we only show the general topologies without indicating specific time orderings. The energy denominators and overall factors in each individual case can be read off from the operators given in [6]. For a detailed discussion of the NLO corrections the reader is referred to [8]. Apart from the explicit M_π-dependence of the OPE in eq. (1) an additional implicit dependence of the ratio $g_{\pi N}/m_N$ on the pion mass has to be taken into account at NLO. For an arbitrary value $\tilde{M}_\pi$ of the pion mass one has

$$\frac{g_{\pi N}}{m_N} = \frac{g_A}{F_\pi}\left(1 - \frac{g_A^2 \tilde{M}_\pi^2}{4\pi^2 F_\pi^2}\ln\frac{\tilde{M}_\pi}{M_\pi} - \frac{2\tilde{M}_\pi^2}{g_A}\bar{d}_{18} + \left(\frac{g_A^2}{16\pi^2 F_\pi^2} - \frac{4}{g_A}\bar{d}_{16} + \frac{1}{16\pi^2 F_\pi^2}\bar{l}_4\right)\left(M_\pi^2 - \tilde{M}_\pi^2\right)\right), \quad (2)$$

where $g_A = 1.26$, $F_\pi = 92.4\,\text{MeV}$ and $M_\pi = 138\,\text{MeV}$ denote the physical values of the nucleon axial vector coupling, pion decay constant and pion mass, respectively. Further, $\bar{l}_4$, $\bar{d}_{18}$ and $\bar{d}_{16}$ are LECs related to pion and pion-nucleon interactions. We use the following values for these LECs: $\bar{l}_4 = 4.3$ [9], $\bar{d}_{16} = -1.23^{+0.32}_{-0.53}\,\text{GeV}^{-2}$ [10] and $\bar{d}_{18} = -0.97\,\text{GeV}^{-2}$. The constant $\bar{d}_{18}$ is fixed from the observed value of the Golberger-Treiman discrepancy with $g_{\pi N} = 13.2$ [11]. Note further that for the LEC $\bar{d}_{16}$ we use an average of three values given in [10], which result from different fits. The shown uncertainty is defined in the way to cover the whole range of values from [10].

The remaining $\tilde{M}_\pi$-dependence of the nuclear force at NLO is given by the TPE [8] as well as by the short-range terms of the form

$$V_{\tilde{M}_\pi}^{\text{cont}} = \tilde{M}_\pi^2\left[\bar{D}_S + \bar{D}_T(\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2) - \left(\beta_S + \beta_T(\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2)\right)\ln\frac{\tilde{M}_\pi}{M_\pi}\right], \quad (3)$$

where the constants $\beta_{S,T}$ are given in terms of g_A, F_π and C_T [8]. All other contact terms do not depend on the pion mass and the corresponding LECs $C_{1,\ldots,7}$ can be adopted from the analysis of [12], performed for the physical value of $\tilde{M}_\pi$. The essential difficulty in extrapolating the nuclear forces in the pion mass is due to the fact that one cannot disentangle the short-range contribution given in eq. (3) from the LO one in eq. (1) in the NN system and at the physical point $\tilde{M}_\pi = M_\pi$[2]. Thus, the LECs D_S and D_T are unknown. In order to proceed further we assume natural values for these constants, *i.e.*:

$$\bar{D}_{S,T} = \frac{\alpha_{S,T}}{F_\pi^2 \Lambda_\chi^2}, \quad \text{where } \alpha_{S,T} \sim 1, \quad (4)$$

and $\Lambda_\chi \simeq 1\,\text{GeV}$. In [13] we have shown that *all values* of the dimensionless coefficients α related to the contact terms lie at NLO in the range $-2.1\ldots 3.2$ for all cut-offs employed. In the following we will make a conservative estimation for $\alpha_{S,T}$:

$$-3.0 < \alpha_{S,T} < 3.0. \quad (5)$$

Certainly, the lack of information about the values of $D_{S,T}$ is the main source of uncertainty of our analysis. We have adopted the same procedure to regularize the LS equation as in [12], *i.e.* the potential $V(\boldsymbol{p}', \boldsymbol{p})$ is multiplied by the regulator functions $f_{\text{R}}^{\text{expon}}(|\boldsymbol{p}|)$, $f_{\text{R}}^{\text{expon}}(|\boldsymbol{p}'|)$, whose precise form is given in [12]. In the next section we will show how various observables behave with $\tilde{M}_\pi$.

3 Results

Having specified the NN interaction at arbitrary value of $\tilde{M}_\pi$ we are now in the position to calculate observables. Let us begin with the behavior of the NN phase shifts in

[2] This can be done in the processes including pions such as, *e.g.*, pion-deuteron scattering. Such an analysis is however not yet available.

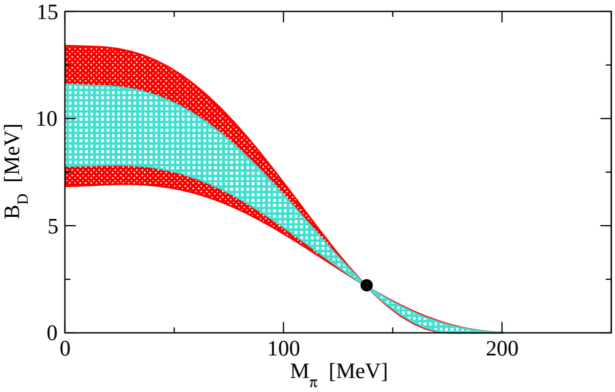

Fig. 2. Deuteron BE *versus* $\tilde{M}_\pi$. The shaded areas show allowed values. The light-shaded band corresponds to our main result with the uncertainty due to the unknown LECs $\bar{D}_{S,T}$. The dark-shaded band gives the additional uncertainty due to the uncertainty of $\bar{d}_{16}$. The heavy dot shows the BE for the physical case $\tilde{M}_\pi = M_\pi$.

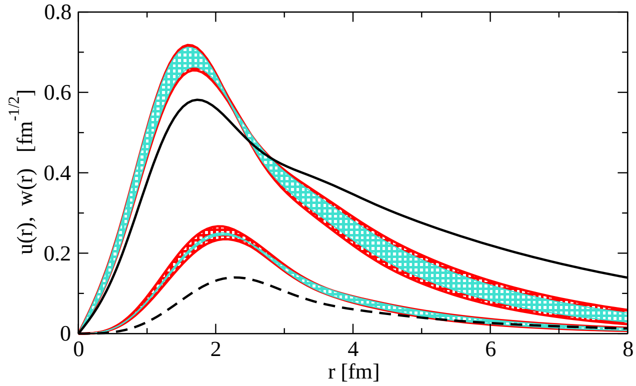

Fig. 3. Deuteron wave functions in the CL compared with the ones in the physical case. The upper and lower bands (solid and dashed lines) refer to the S- and D-wave functions $u(r)$ and $w(r)$ in the CL (in the physical case), respectively. For remaining notations see fig. 2.

the CL. First of all we would like to stress that the OPE in eq. (1) leads to a significant scattering even in higher partial waves due to the Coulomb-like pion propagator in the CL. Further, no effective range expansion of the form

$$k^{2l+1} \cot \delta_l(k) = -\frac{1}{a_l} + r_l \frac{k^2}{2} + v_l^2 k^4 + \cdots , \qquad (6)$$

exists due to the vanishing pion mass[3]. In eq. (6) k is the c.m. momentum and l the angular momentum. It is easy to derive the low-momentum behavior of $\delta_l(k)$ at least for large l, where the potential becomes weak and one can use the Born approximation for the T-matrix. It is then sufficient to look at on-the-energy shell matrix elements of the OPE in the CL $V_{\rm OPE}^{\rm CL}(k)$, which strongly dominantes the nuclear interaction at low momenta. We found that $V_{\rm OPE}^{\rm CL}(k) = 0$ in all spin-singlet ($s = 0$) and $V_{\rm OPE}^{\rm CL}(k) = \gamma$ in the spin-triplet ($s = 1$) channels, where the constant γ depends on the partial wave. As a consequence, we expect in the CL a strong reduction of $\delta_l(k)$ in the $s = 0$ channels and linear with k growth of $\delta_l(k)$ in the $s = 1$ channels. Numerical analysis performed in [8] confirms this estimation. It remains to stress that we have not found new bound states in the CL in agreement with the previous work by Bulgac *et al.* [14], although a strong enhancement of $\delta_l(k)$ is observed in many cases[4]. Last but not least, we found smaller (in magnitude) and more natural values for the two S-wave scattering lengths in the chiral limit $a_{\rm CL}(^1S_0) = -4.1 \pm 1.6^{+0.0}_{-0.4}$ fm and $a_{\rm CL}(^3S_1) = 1.5 \pm 0.4^{+0.2}_{-0.3}$ fm, where the first indicated error refers to the uncertainty in the value of $\bar{D}_{^3S_1}$ and $\bar{d}_{16}$ being set to the average value $\bar{d}_{16} = -1.23\,\mathrm{GeV}^{-2}$, while the second indicated error shows the additional uncertainty due to the uncertainty in the determination of $\bar{d}_{16}$ as described before.

We have also calculated the deuteron binding energy (BE) as a function of $\tilde{M}_\pi$. Our results for the cut-off $\Lambda = 560\,\mathrm{MeV}$ are depicted in fig. 2. According to our *complete NLO analysis* deuteron is stronger bound in the chiral limit with the BE $B_{\rm D}^{\rm CL} = 9.6 \pm 1.9^{+1.8}_{-1.0}\,\mathrm{MeV}$. For the root-mean-square radius of the deuteron in the CL we found a smaller value $r_{\rm D}^{\rm CL} = 1.27 \pm 0.09 \pm 0.04\,\mathrm{fm}$ to be compared with the observed one $r_{\rm D} = 1.97\,\mathrm{fm}$. A more short-range nature of the deuteron in the CL is also clearly visible in the deuteron wave function presented in fig. 3. It is interesting that the probability for the deuteron to be in the D-state is strongly enhanced in the CL ($9.5\% \ldots 11.8\%$ compared to 3.5% for the NLO with the observed value of the pion mass). The detailed description of the deuteron properties will be published elsewhere.

4 Summary

To conclude, we did not find dramatic changes in the properties of the NN systems in the CL, such as the appearance of new bound states. Various observables like the deuteron binding energy and the S-wave scattering lengths are shown to be more natural in the CL.

[3] The maximal radius of convergence of the effective range expansion is proportional to $\tilde{M}_\pi^2$ and goes to zero in the CL.

[4] For example, $\delta_l(k)$ in the $^3P_0$ partial wave reaches a maximum of $\sim 32^\circ$ to be compared with $\sim 11^\circ$ in the physical case.

References

1. J. Gasser, H. Leutwyler, Phys. Rep. C **87**, 77 (1982).
2. S.R. Beane *et al.*, Nucl. Phys. A **700**, 377 (2002).
3. M. Fukugita *et al.*, Phys. Rev. D **52**, 3003 (1995).
4. S.R. Beane, M.J. Savage, Nucl. Phys. A **720**, 399 (2003), hep-ph/0206113.
5. S. Weinberg, Nucl. Phys. B **363**, 3 (1991).

6. E. Epelbaoum, W. Glöckle, Ulf-G. Meißner, Nucl. Phys. A **637**, 107 (1998).
7. S. Okubo, Prog. Theor. Phys. Jpn. **12**, 603 (1954).
8. E. Epelbaum, Ulf-G. Meißner, W. Glöckle, nucl-th/0207089.
9. J. Gasser, H. Leutwyler, Ann. Phys. (N.Y.) **158**, 142 (1984).
10. N. Fettes, doctoral thesis, published in *Berichte des Forschungszentrum Jülich*, **3814** (2000).
11. E. Matsinos, hep-ph/9807395.
12. E. Epelbaum, W. Glöckle, Ulf-G. Meißner, Nucl. Phys. A **671**, 295 (2000).
13. E. Epelbaum, Ulf-G. Meißner, W. Glöckle, Ch. Elster, Phys. Rev. C **65**, 044001 (2002).
14. A. Bulgac, G.A. Miller, M. Strikman, Phys. Rev. C **56**, 3307 (1997).